

#### AN INVENTORY MODEL FOR DETERIORATING ITEMS WITH SHORTAGES AND INFLATION UNDER TWO STORAGE FACILITY Shruti Jindol<sup>\*1</sup> & Dr. S. P. Singh<sup>2</sup>

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### ABSTRACT

In the present paper an inventory model is developed for deteriorating items under two storage capacity facilities. In many commercial activities there are the various reasons which forced the buyer to order more than the warehouse capacity. Such situation for additional storage space called rent ware house. For the excess quantity ware rent ware house is used. Demand rate is assumed to be a stock and time dependent with Non-instantaneous Time dependent deterioration rate. The methodology has been given to find out the optimal cycle time and ordering quantity with the total cost. Shortages at the owned warehouse are also allowed subject to partial backlogging. All cost components are affected by the both inflation and time value of money. Inflation plays major role it increases the cost of the goods. The solution methodology provided in the model helps to decide on the feasibility of renting a warehouse. Finally, findings have been illustrated with the help of numerical examples. Comprehensive sensitivity analysis has also been provided.

Keywords: Inventory; Warehouse; Deterioration; Partial Backlogging; Lost Sales; Inflation

### 1. INTRODUCTION

In the past, many researchers worked on inventory problems for deteriorating items such as medicines, Deterioration means decay, spoilage, damage out off trend, evaporation. Deteriorating items like fruits and vegetables, volatile liquids, blood, fashion goods etc. seasonal products and many others. Deteriorating inventory systems have first introduced by Ghare and Scharder (1967). Their work was extended by Covert and Philip (1973) by introducing a variable rate of deterioration. Then many authors by Shah (1977) by considering a model allowing complete backlogging of the unsatisfied demand. Dave and Patel (1981) considered for deteriorating items with time proportional demand and shortages. Kang and Kim (1983) study on the price and production level of the deteriorating inventory system. Datta and Pal (1988) developed an EOQ model by introducing a variable deterioration rate and power demand pattern. Aggarwal and Jaggi (1989) considered an ordering policy for decaying inventory. Shiue (1990), Hariga and Benkherouf (1994) developed an for deteriorating items with exponential time-varying demand. Then Hariga (1995) extended this work to allow shortages. So many researchers, namely, Chakrabarti and Chaudhuri (1997), Wee (1999) and Papachristos and Skouri (2003) continued their research in the area of inventory management for deteriorating items in various situations. In real life there are many items which start to deteriorate after their maximum life time i.e. non- instantaneous deterioration. Manna and Chaudhuri (2006), Skouri et al. (2009) and Wu, Ouyang and Yang (2009) have focused on non-instantaneous deterioration rate.

In classical inventory models it is assumed that warehouse has infinite storage space. But it is not feasible according to real life situations, so it is usually assumed that organizations own a warehouse (OW) with limited storage space. It is generally seen that enterprises purchase more goods than can be held in their owned warehouses (OW) for many reasons, such as discounts on bulk purchases, etc. The excess units are stored in an additional storage space. This additional storage space may be a rented warehouse (RW). The holding cost in the RW is generally assumed to be higher than that in the OW If quantity exceeds the limited storage space of OW then organizations hire a rented warehouse (RW) with abundant storage space and cost for storing is greater than the storing cost of OW. To reduce the inventory costs, it is imperative to consume the goods of the RW at the earliest time .As a result, the stocks of the OW will not be released until the stocks of the RW are exhausted. A two warehouse model was considered by **Hartely (1976)** under the assumption that the holding cost in the RW is greater than that in the OW. **Sarma (1987)** have taken first step in the direction of limited storage capacity of OW. He has developed a two- warehouse inventory model for deteriorating items and limited storage capacity. **Chaudhuri (1992)** further developed the model with or without shortages with the



assumption that the demand varies over time with a linearly increasing trend and that the transportation cost from the RW to the OW depends on the quantity being transported. After that many authors have studied twowarehouse problems Pakkala and Achary (1992a,b) further considered the two-warehouse model for deteriorating items with finite replenishment rate and shortages, taking time as a discrete and continuous variable, respectively. Subsequently, many authors such as Bhunia and Maiti (1994, 1998), Kar, Bhunia, and Maiti (2001), Zhou and Yang (2003), Yang (2004, 2006), Lee (2006), Chung and Huang (2007), Das, Maity, and Maiti (2007), Dye, Ouyang, and Hsieh (2007), Niu and Xie (2008), Rong, Mahapatra, and Maiti (2008) and many other like Kumari, Singh and Kumar (2008), Singh, Kumar and Kumari (2010), Singh, Kumari and Kumar (2010, 2011), Singh, Jain and Pareek (2012), Singh, Gupta and Gupta (2013) and Ghiami et al. (2013) etc. have worked in the area of two-warehousing under different scenarios. Before 1970's inflation is disregarded by researchers. After that it is observed that many countries are suffered from inflation and time value of money. But in real life, the impact of inflation cannot be ignored while deciding the optimal inventory policies. Buzacott(1975) developed an EOQ model under the impact of inflation. Misra (1975) considered the effect of inflationary conditions on inventory systems. Bierman and Thomas (1977) proposed the EOQ model considering the effect of both inflation and time value of money. We can find other interesting ideas in Dye et al. (2008), Hsieh and Dye (2010) etc.

Demand is an important factor for any type of inventory systems. Stock dependent demand rate is first introduced by Liao et al. (2000). They have developed an inventory model with initial stock consumption rate and permissible delay in payment. For detailed study we can go through the work of many other authors like Yadav, Singh and Kumari (2012) etc. Due to excess2demand stock level reaches at zero. In such conditions suppliers try to retain the customers for this they have considered the partially backlogged shortages Singh and jindal (2016) discussed a model for multiple market demand in production inventory model. Shortage conditions have been discussed by several researchers like Pentico and Drake (2011), Singh and Saxena (2012, 2013), Singh et al. (2013), Taleizadch and Pentico (2013) and Ghiami et al. (2013).

In this paper we have developed a two warehouse inventory model with limited storage space in OW for noninstantaneous deteriorating items with stock dependent demand rate in an inflationary environment. Shortages are allowed and partially backlogged and backlogging rate is inversely proportional to the waiting time. At the end a numerical analysis and sensitivity analysis are given.

### 2. ASSUMPTIONS AND NOTATIONS

Following assumptions and notations are used in mathematical model formulation; **Assumptions:** 

- a. Demand rate is stock dependent and taken as following form
- b. D (I(t)) =  $\begin{cases} \alpha + \beta I(t) & \text{if } I(t) > 0; \\ \alpha & \text{if } I(t) \le 0; \end{cases}$

- c. Shortages are allowed and partially backlogged where backlogging rate is d.  $B(t) = \frac{1}{1+\delta t}$ ; where t is the waiting time and  $0 < \delta < 1$  is the backlogging parameter.
- Model is dealing with single non- instantaneous deteriorating item. There is no deterioration during e. time period [0, t<sub>d</sub>] and deterioration occurs during time interval [t<sub>d</sub>, t<sub>2</sub>] with deterioration rate  $\theta(t) = \theta$  t, where  $0 < \theta <<1$  is deterioration parameters.
- Time horizon is infinite and replenishment rate is infinite with zero lead time. f
- The Owned Warehouse (OW) has limited space of  $W_2$  units where as the Rented Warehouse has g. unlimited space area.
- The holding cost  $(h_1)$  of RW is greater than the holding cost  $(h_2)$  of OW. Therefore consumptions of h. inventory starts only when inventory level of RW researches zero.
- The charges for transportation as well as time between RW and OW are negligible. i.

### **Notations:**

D (I(t)): Instantaneous stock level dependent demand rate;

- 0 The order quantity; :
- Non-instantaneous Time dependent deterioration rate;  $\theta(t)$

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r	:	Difference of inflation and time discounting;				
$h_1$	:	The holding cost (per unit per time unit) of Rented warehouse (RW);				
$h_2$	:	The holding cost (per unit per time unit) of Owned warehouse (OW);				
А	:	Ordering cost; C: The unit purchasing cost; $C_s$ : The shortage cost; $C_L$ : The				
		lost sale cost;				
$W_2$	:	The limited space area of OW; $W_1$ : Maximum inventory level in RW;				
В	:	Maximum backorder level				
t <sub>d</sub>	:	The maximum life time of an item;				
t <sub>1</sub>	:	The time period at which inventory level in RW researches at zero;				
t <sub>2</sub>	:	The time period at which inventory level in OW researches at zero;				
Т	:	The Cycle length;				
$I_1(t)$	:	The Inventory level in RW during time period $[0, t_d]$ ;				
$I_2(t)$	:	The Inventory level in RW during time period $[t_d, t_1]$ ;				
$I_3(t)$	:	The inventory level in OW during time period $[0, t_d]$ ;				
$I_4(t)$	:	The inventory level in OW during time period $[t_d, t_1]$ ;				
$I_5(t)$	:	The Inventory level in OW during time period $[t_1, t_2]$ ;				
$I_6(t)$	:	The Inventory level in OW during time period $[t_2, T]$ ;				
TRC	:	The Present worth of total relevant cost;				
PC	:	The purchase cost; HC: The Present worth of holding cost; SC: The Present				
		worth of Shortage cost: LC: The Present worth of lost sale cost:				

### 3. MATHEMATICAL MODEL FORMULATION

After satisfying the backlogged shortages of previous period the inventory level at time t = 0 is S out of which W units are stored in OW and remaining S- W units are stored in RW. In this paper we have considered the case:  $t_d \le t_1$ .

### Case: when $(t_d \le t_1)$

In this case, Inventory level of RW decreases due to demand during the time interval  $[0, t_d]$ . At time  $t = t_d$  deterioration occurs. During  $[t_d, t_1]$  inventory level decreases due to the combine effect of demand and deterioration and at  $t = t_1$  it reaches at zero level. After time  $t_1$  demand of items is fulfilled by using inventory of OW during time interval  $[t_1, t_2]$ . In the time period  $[t_2, T]$  shortages occurs and partially backlogged. The description of this case is presented in Figure 1.





The inventory depletion in RW is represented by differential equation given as below

$$\frac{dI_1(t)}{dt} = -(\alpha + \beta I_1(t)) \qquad \qquad 0 \le t \le t_d$$
(1)
$$\frac{dI_2(t)}{t_t} + t\theta I_2(t) = -(\alpha + \beta I_2(t)) \qquad \qquad t_d \le t \le t_1$$

And in OW inventory functioning is as follows

$$I_{3}(t) = W_{2}$$

$$0 \le t \le t_{d}$$

$$dI_{4}(t) = tOI_{4}(t)$$

$$t_{d} \le t \le t_{d}$$

$$\frac{d}{dt} = -t \Theta I_4(t) \qquad \qquad t_d \le t \le t_1$$
(4)

$$\frac{dI_5(t)}{dt} + t\theta I_5(t) = -(\alpha + \beta I_5(t)) \qquad \qquad t_1 \le t \le t_2$$

$$\frac{dI_6(t)}{dt} = -\left(\frac{\alpha}{\left(1 + \delta(T - t)\right)}\right) \qquad t_2 \le t \le T$$
(6)

 $\begin{array}{l} \text{Under the boundary conditions } I_1(t=0)=W_1, \ I_1(t_d)=I_2(t_d) \ , \ \ I_2(t_1)=0, \ I_3(t_d)=I_4(t_d)=W_2, \ I_5(t_2)=0, \ I_6(t_2)=0, \ I_6(T)=-B. \end{array}$ 

Now solving above equations we get

$$I_{1}(t) = \left[\frac{\alpha}{\beta}\left(e^{\beta(t_{d}^{-t)}} - 1\right) + \alpha e^{-(t\beta + \theta^{\frac{t^{2}}{2}})}\left\{\left(t_{1} - t_{d}\right) + \beta\left(\frac{t_{1}^{2} - t_{d}^{2}}{2}\right) + \theta\left(\frac{t_{1}^{3} - t_{d}^{3}}{6}\right)\right\}\right]$$

$$(7)$$

$$I_{2}(t) = \left[e^{-(t\beta + \theta^{\frac{t^{2}}{2}})}\left\{\left(t_{1} - t\right) + \beta\left(\frac{t_{1}^{2} - t^{2}}{2}\right) + \theta\left(\frac{t_{1}^{3} - t^{3}}{6}\right)\right\}\right]$$

$$(8)$$

$$I_{4}(t) = W_{2}e^{-\theta^{\left(\frac{t^{3}}{2} - t^{2}\right)}}$$

$$(9)$$

$$I_{5}(t) = e^{-(t\beta + \theta^{\frac{t^{2}}{2}})}\left\{\left(t_{2} - t\right) + \beta\left(\frac{t_{2}^{2} - t^{2}}{2}\right) + \theta\left(\frac{t_{2}^{3} - t^{3}}{6}\right)\right\}$$

$$(10)$$

$$I_{6}(t) = -\alpha(t - t_{2})$$

$$(11)$$

$$W_{1} = \left[\frac{\alpha}{\beta}\left(e^{\beta t_{d}} - 1\right) + \alpha e^{-(\theta^{\frac{t^{2}}{2}})}\left\{\left(t_{1} - t_{d}\right) + \beta\left(\frac{t_{1}^{2} - t_{d}^{2}}{2}\right) + \theta\left(\frac{t_{1}^{3} - t_{d}^{3}}{6}\right)\right\}\right]$$

$$(12)$$

From continuity we have  $I_4(t_1) = I_5(t_1)$ , Hence

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$$t_{1} = (-\frac{1}{2\theta W_{2}})(-6\alpha - 3\beta W_{2} + \theta t_{2}W_{2} \pm \{(6\alpha + 3\beta W_{2} - \theta t_{2}W_{2})^{2} + 4\theta W_{2} - 6\alpha t_{2} + 6W_{2} - 3\beta t_{2}W_{2} - t^{2}{}_{2}W_{2}\theta + 3t^{2}{}_{d}W_{2}\theta\}^{\frac{1}{2}}$$

Maximum backlogged amount IB = -I<sub>6</sub>(t = T),  $B = \alpha(T - t_2)$ Order quantity Q = W<sub>1</sub>+W<sub>2</sub>+ B

$$Q = \frac{\alpha}{\beta} (e^{\beta t} d - 1) + \alpha e^{-(\theta - \frac{t}{2})} \{ (t_1 - t_d) + \beta \left( \frac{t_1^2 - t_d^2}{2} \right) + \theta \left( \frac{t_1^3 - t_d^3}{6} \right) \} + W_2 + \alpha (T - t_2)$$
(13)

The total relevant cost includes following cost parameters

- a. The ordering cost = A
- b. The purchasing cost = CQ
- c. The Present worth of holding cost (HC) is

$$\begin{split} HC &= h_{1} \{ \int_{0}^{t} I_{1}(t)e^{-rt} dt + \int_{td}^{t} I_{2}(t)e^{-rt} dt \} + h_{2} \{ \int_{0}^{t} I_{3}(t)e^{-rt} dt + \int_{td}^{t} I_{4}(t)e^{-rt} dt \int_{t}^{t} I_{5}(t)e^{-rt} dt \} \\ HC &= h_{1} \{ \alpha \left[ \frac{r_{d}^{2}}{2} - \frac{r_{d}^{3}}{6} \right] + \alpha \left[ t_{d} - \frac{(\beta + r)t_{d}^{2}}{2} + \frac{(2r\beta - 3\theta)t_{d}^{3}}{6} + \frac{(\theta r)t_{d}^{4}}{4} \right] \{ (t_{1} - t_{d}) + \beta \left[ \frac{t_{1}^{2} - t_{d}^{2}}{2} \right] + \theta \left[ \frac{t_{1}^{3} - t_{d}^{3}}{6} \right] \} \\ &+ \alpha \{ t_{1} \} + \beta \left[ \frac{t_{1}^{2}}{2} \right] + \theta \left[ \frac{t_{1}^{3}}{6} \right] \} \{ t_{1} - t_{d} \} - \beta \left[ \frac{t_{1}^{2} - t_{d}^{2}}{2} \right] - \theta \left[ \frac{t_{1}^{3} - t_{d}^{3}}{6} \right] \} - \alpha \{ \left[ \frac{t_{1}^{2} - t_{d}^{2}}{2} \right] - (\beta + r) \left[ \frac{t_{1}^{3} - t_{d}^{3}}{3} \right] - \theta \left[ \frac{t_{1}^{3} - t_{d}^{3}}{6} \right] \} - \beta \{ \left[ \frac{t_{1}^{3} - t_{d}^{3}}{6} \right] \} - \beta \{ \left[ \frac{t_{1}^{3} - t_{d}^{3}}{6} \right] - (\beta + r) \left[ \frac{t_{1}^{3} - t_{d}^{3}}{6} \right] - \theta \{ \left[ \frac{t_{1}^{3} - t_{d}^{3}}{6} \right] \} - \theta \{ \left[ \frac{t_{1}^{3} - t_{d}^{3}}{6} \right] - (\beta + r) \left[ \frac{t_{1}^{5} - t_{d}^{5}}{30} \right] - \theta \{ \left[ \frac{t_{1}^{3} - t_{d}^{3}}{6} \right] \} + \alpha \{ t_{1} + t_{d} + t$$

d. The Present Worth of shortage cost (SC) is

$$SC = C_s \int_{t_2}^{T} I_6(t) e^{-rt} dt = C_s \alpha t_2 ((r \frac{T^2 - t_2^2}{2} - (T - t_2)) + (\frac{T^2 - t_2^2}{2} - r \frac{T^3 - t_2^3}{3}))$$

(15)

e. The Present Worth of Lost sale cost (LC) is

$$LC = C_L \alpha \int_{t_2}^{T} (1 - \frac{1}{(1 + \delta(T - t))}) e^{-rt} dt$$

$$LC = C_L (\alpha(T - t_2) - r \frac{T^2 - t_2^2}{2} - \frac{1}{\delta} \log(1 + \delta(T - t_2)) + \frac{r}{\delta} (-(\delta T + 1) \log(1 + \delta(T - t_2) - (T - t_2)))$$
(16)

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TRC (
$$t_1, t_2, T$$
) = (1/T)[A + HC + PC + SC + LC]  
(17)

To minimize total relevant cost, we differentiate K= TC  $(t_1, t_2, T)$  w. r. t to  $t_1, t_2$  and T and for optimal value necessary conditions are  $\frac{\partial TC(t_1, t_2, T)}{\partial t_1} = 0; \frac{\partial TC(t_1, t_2, T)}{\partial t_2} = 0; \frac{\partial TC(t_1, t_2, T)}{\partial T} = 0;$ 

Provided the determinant of principal minor of hessian matrix are positive definite, i.e. det(H1)>0, det(H2)>0,det(H3)>0 where H1, H2, H3 is the principal minor 0f the Hessian-matrix.

#### Hessian Matrix of the total cost function is as follows:

$\left[\frac{\partial^2 TC}{2}\right]$	$\frac{\partial^2 TC}{\partial t}$	$\frac{\partial^2 TC}{\partial t \partial T}$
$\left  \begin{array}{c} \partial t_1^2 \\ \partial^2 TC \end{array} \right $	$\partial^2 TC$	$\frac{\partial^2 TC}{\partial^2 TC}$
$\overline{\partial t_2 \partial t_1}$	$\partial t_2^2$	$\overline{\partial t_2 \partial T}$
$\frac{\partial^2 TC}{\partial T \partial t_1}$	$\frac{\partial^2 TC}{\partial T \partial t_2}$	$\frac{\partial^2 TC}{\partial T^2}$

### 4. NUMERICAL EXAMPLE

For the Illustration of proposed model we consider following inventory system in which values of different parameters in proper units are

 $A = 200, \alpha = 195, \beta = 20, C = 2, C_s = 1.6, C_L = 1.5, t_d = 0.02, \theta = 0.05, \delta = 0.1, r = 0.02, h_1 = 0.3, h_2 = 0.2, W = 50$ 

Using Mathematical software Mathematica 6 we get the optimal values of  $t_1^* = 0.0453294$ ,  $t_2^* = 0.0815296$ ,  $T^* = 1.22862$ ,  $Q^* = 240.289$ ,  $K^* = 452.834$ 

### 5. SENSITIVITY ANALYSIS

The study of effect of change of different parameters on total relevant cost, a sensitivity analysis is performed by varying some parameters like demand parameters ' $\alpha$ ' ' $\beta$ ', deterioration rate $\theta$ , inflation rate r, etc. We have changed the values of one parameter at a time and taking other parameters with their original values, given in above numerical example and resulting values are given in table 1.



Fig. 2 Convexity of K<sup>\*</sup> (Total relevant cost) w. r. t. T<sup>\*</sup> and t<sub>2</sub><sup>\*</sup>



# International Journal of Engineering Researches and Management Studies Table 1 (Case- 1)

Variation in constant demand factor $\alpha$									
Variations in $\alpha$	$t_1^*$	$t_2^*$	$T^*$	Q*	K <sup>*</sup>				
185	0.0453455	0.0817013	1.25828	226.395	437.985				
190	0.0453373	0.081615	1.24314	229.91	445.436				
200	0.0453218	0.0814451	1.21465	233.377	460.181				
205	0.0453145	0.0813614	1.20122	236.833	467.478				
Variation in stock dependent demand factor $\beta$									
Variations in $\beta$	$t_1^*$	$t_2^*$	$T^*$	Q*	K <sup>*</sup>				
10	0.0881431	0.152835	1.30747	265.672	476.809				
15	0.0598953	0.106571	1.2587	249.327	461.969				
25	0.036456	0.0659903	1.20851	234.539	446.741				
30	0.0304863	0.0554373	1.10419	230.555	442.402				
Variation in maximum life time t <sub>d</sub>									
Variations in t <sub>d</sub>	t <sub>1</sub> *	t <sub>2</sub> **	<u>T</u> *	<u>Q</u> *	K <sup>*</sup>				
0.01	0.0453351	0.0815016	1.24539	247.992	457.816				
0.025	0.0453268	0.0815399	1.22133	236.893	450.668				
0.03	0.0453243	0.0815478	1.21479	233.641	448.726				
0.035	0.045322	0.0815531	1.20904	230.538	447.016				
Variation in deteri	oration parameter $\theta$	*	*	•*	*				
Variations in $\theta$	t <sub>1</sub>	t <sub>2</sub>	T 1 22065	Q	<u>K</u>				
0.04	0.0453316	0.0815615	1.22865	240.291	452.846				
0.02	0.0453305	0.0815455	1.22864	240.29	452.84				
0.03	0.0453284	0.0815137	1.2286	240.287	452.829				
0.04	0.0453273	0.0814978	1.22858	240.285	452.823				
Variation in partia	l backlogging parameter δ	. *	<b>–</b> *	~* 	***				
Variations in $\delta$	$t_1$	$t_2$	1 22856	<u>Q</u> 240.291	<u>K</u>				
0.05	0.0453141	0.0815030	1.22850	240.281	455.45				
0.15	0.0453545	0.0815382	1.22803	240.29	452.745				
0.1	0.045337	0.0815424	1.22864	240.29	452.698				
0.25	0.0453384	0.0815448	1.22864	240.29	452.671				
Variation in r									
	$l_1$	$\frac{l_2}{0.0800200}$	1 21822	<u>Q</u> 228 271	<u> </u>				
0.01	0.0452009	0.0809399	1.21852	238.371	455.941				
0.015	0.0452648	0.0812342	1.22341	241.32	453.391				
0.025	0.0453949	0.0818261	1.23393	242.277	452.272				
0.03	0.0454612	0.0821237	1.23935	243.287	451.702				
Variation in UW capacity W       Variations in W $t^*$ $V^*$									
	0.0453259	$\frac{12}{0.081/1267}$	1 22837	240.256	<u>152 761</u>				
	0.0452277	0.0014207	1.22037	240.250	452.701				
4J 55	0.0453277	0.0014/02	1.22049	240.272	452.171				
55 60	0.0453312	0.081581	1.228/4	240.305	452.8/1				
b0         0.045333         0.0816324         1.22886         240.321         452.908           Keen absorvation of all above table 1 reveals following facts         Sector         Contract of all above table 1 reveals following facts									

Keen observation of all above table-1 reveals following facts

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- Increase in  $\alpha$  results in decrement in  $t_1^*, t_2^*, T^*$  while increment in  $K^*, Q^*$ . i.
- ii.
- Increase in  $\beta$  results in decrement in  $t_1^*, t_2^*, T^*, K^*, Q^*$ . Increase in  $t_d$  results in decrement in  $t_1^*, T^*, Q^*, K^*$  while increment in  $t_2^*$ . Increase in  $\theta$  results in decrement in  $t_1^*, t_2^*, T^*, K^*, Q^*$ . iii.
- iv.
- Increase in  $\delta$  results in decrement in  $T^*$ ,  $Q^*$ ,  $K^*$  while increment in  $t_1^*$ ,  $t_2^*$ . v.
- Increase in r results in decrement in K<sup>\*</sup> while increment in  $t_1^*, t_2^* T^*, Q^*$ . vi.
- Increase in W results in increment in  $t_1^*, t_2^*, T^*, K^*, Q^*$ . vii.

### 6. CONCLUSION

In this paper we have studied two- warehouse inventory model for deteriorating items with non- instantaneous deterioration rate. Demand rate is stock dependent in an inflationary environment and shortages are allowed and partially backlogged with inverse backlogging rate. We have optimized the total relevant cost and illustrated this model numerically. To check the sensitivity of the model we have performed a sensitivity analysis by changing values of major parameters. Our model is applicable for the fruits and vegetables, bakery products etc. This model can be extended by incorporating other parameters of inventory control system.

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